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A classical manifestation of the Pauli exclusion principle

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Abstract: The occupied and unoccupied fermionic BPS quantum states of a type-IIA string stretched between a D6-brane and an orthogonal D2-brane are described in M-theory by two particular holomorphic curves embedded in a Kaluza–Klein monopole. The absence of multiply-occupied fermionic states — the Pauli exclusion principle — is manifested in M-theory by the absence of any other holomorphic curves satisfying the necessary boundary conditions. Stable, non-BPS states with multiple strings joining the D6-brane and D2-brane are described M-theoretically by non-holomorphic curves.

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A striking aspect of duality symmetries is that they exchange classical and quantum properties of the system under study. For instance T-duality exchanges the momentum, $n\hbar/R$, of a closed string with the energy stored in winding, $2\pi n T_F R$, where T_F is the fundamental string tension. The quantization of winding is a classical geometric property of large smooth strings while the quantization of momentum is a consequence of the single-valuedness of the quantum mechanical wave function. T-duality exchanges these classical and quantum features. Such relationships between classical and quantum phenomena should arise naturally in the M-theory description of string theory.

One principle that appears to be essentially quantum mechanical is the Pauli exclusion principle, which forbids two fermions from occupying the same quantum state. In this note we will point out how this can arise classically in an M-theory context. We will consider an M-theory membrane in the background of a Kaluza-Klein monopole, which is the M-theory description of a six-brane. The membrane is placed so as to be asymptotically transverse to the six-brane. In the type IIA limit this configuration is described by a D6-brane and an orthogonal D2-brane spaced a finite distance apart. The configuration has a total of eight (DN) directions that are in one brane and transverse to the other and it preserves one quarter of the 32 components of the M-theory supersymmetry. This setup is very special since a fundamental string stretching between the two D-branes has a unique, localized BPS state which is necessarily a fermion. As a result there are only two independent BPS quantum states in the second-quantized theory — the fermionic string is either present or absent. In the M-theory description of this system each of these configurations is described by a single smooth membrane which must be holomorphically embedded in the Kaluza–Klein monopole in order for the BPS condition to be satisfied. In this particular context the Pauli exclusion principle will turn out to be the statement that there are only two such holomorphic curves that satisfy the boundary conditions.

Solitonic p-branes that are fermions usually arise from the quantization of zero modes and are part of a supermultiplet which also includes some bosonic states. Furthermore they are usually free to move in one or more spatial directions, and therefore have an infinite number of allowed quantum states. In such circumstances the Pauli exclusion principle would not normally have a direct geometric meaning. The configuration under discussion here is exceptional because the membrane dual to the fundamental string is necessarily fermionic and is localized.

The same statement is true for any system related to this by S or T dualities. In fact, a system of this type was used to understand two of the subtle rules of gauge field theory engineering with branes [1] — the anomalous creation of branes and the so-called s-rule which forbids certain supersymmetric configurations of branes from existing. Brane creation can be explained in terms of the anomalous inflow of charge [2] (see also [3, 4, 5, 6] for related arguments) and in geometric terms [7, 8, 9, 10]. The s-rule can be argued [11] to be a consequence of the Pauli exclusion principle. This note builds on the interrelationship between these observations.

The type IIA configuration of interest has a D6-brane along the directions $(x_0, x_1, x_2, x_3, x_4, x_5, x_6)$, and a D2-brane along (x_0, x_7, x_8). The two are separated in the

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x_9 direction. From the M-theory point of view the D6-brane is a Kaluza–Klein monopole described by the metric,

$$ds^2 = -(dx_0)^2 + \sum_{i=1}^{6}(dx_i)^2 + V\, d\vec{r}\cdot d\vec{r} + V^{-1}(dx_{10} + \vec{A}\cdot d\vec{r})^2 \ , \tag{1}$$

where

$$V = 1 + \frac{1}{2|\vec{r}|} \ . \tag{2}$$

Here $x_{10} = x_{10} + 2\pi$ is the angular coordinate, $\vec{r} \equiv (x_7, x_8, x_9)$, and $\vec{A}$ is the monopole field which satisfies the equation $\vec{\nabla}\times\vec{A} = \vec{\nabla}V$. Length scales are measured in units of the radius of the circle on which M-theory is compactified, and which has been set equal to one. The monopole is located at $\vec{r} = 0$.

The Taub-NUT space has three covariantly-constant complex structures. One particular choice of holomorphic coordinates is [9][10]

$$v = x_7 + ix_8 \ , \tag{3}$$

and

$$w = e^{-(x_9 + ix_{10})}\left(-x_9 + \sqrt{x_9^2 + |v^2|}\right)^{1/2} \ . \tag{4}$$

In these coordinates the metric takes the Kähler form

$$ds^2 = -(dx_0)^2 + \sum_{i=1}^{6}(dx_i)^2 + V\, dv d\bar{v} + V^{-1}\left|\frac{dw}{w} - f dv\right|^2 \ , \tag{5}$$

where

$$f = \frac{x_9 + |\vec{r}|}{2|v||\vec{r}|} \ . \tag{6}$$

The metric is smooth everywhere except for a coordinate singularity along the positive x_9-axis.

Any holomorphic curve $w(v)$ describes a BPS configuration of a membrane in this metric. Two simple curves are

$$\text{(a)} \qquad w = e^{-b} \ , \qquad \text{(b)} \qquad w = e^{-b}v \ , \tag{7}$$

with b an arbitrary real constant. The first curve describes a membrane at fixed x_{10}, while in the second x_{10} winds by 2π as v circles clockwise around the origin. The radial profiles of these two curves are mirror reflections of each other,

$$|v| = e^{\pm(x_9 - b)}\sqrt{\pm 2x_9 + e^{\pm 2(x_9 - b)}} \ , \tag{8}$$

where the plus sign corresponds to the first curve, and the minus sign to the second.

The type IIA limit is reached by taking $|x_9|, |v|$ and $|b|$ to infinity, while keeping their ratios finite. This corresponds to blowing up all scales compared to the radius of the compactification circle. For $|v|$ much larger than all other scales, x_9 approaches b

asymptotically, modulo logarithmically-small corrections. Both curves therefore describe a planar D2-brane located at $x_9 \simeq b$. However, the two curves differ drastically in the region $v \sim 0$, where the membrane intersects the x_9 axis. Their behaviour there depends on the sign of b. For example, if b is positive then $x_9 \simeq 0$ for curve (a), while $x_9 \simeq b$ for curve (b), in this region. Therefore, when $b > 0$ the first curve describes a D2-brane located to the right of the D6-brane and attached to it by a stretched string. On the other hand the second curve describes a D2-brane on the right of the transverse D6-brane without any strings attached to it. When $b < 0$ the D2-brane is to the left of the D6-brane and the roles of two curves are interchanged. Now it is curve (b) that describes the state with a string joining the branes while curve (a) describes the disconnected branes.

This geometry of one of the two curves (curve (b)) was described in detail in [9, 10], where it was related to the M-theory description of the phenomenon of anomalous brane creation. However, both curves are needed to make the connection to brane creation in the IIA theory complete. Following the anomaly inflow discussion in [2] the process of brane creation should be viewed as a phenomenon in which second quantization plays an essential rôle. In the region $b < 0$, and in the type IIA limit, a first quantized string stretching between the D-branes has a negative-energy fermionic level. Curve (a) describes the state in which this level is occupied, while curve (b) describes the state in which this level is empty. In a second-quantized formalism curve (a) thus describes the vacuum state of the Dirac sea, in which all negative energy levels are occupied and positive levels empty, while curve (b) describes a physical hole excitation. As b is varied adiabatically, the first-quantized energy levels shift uniformly, just as they do in compact $(1+1)$-dimensional QED when an electric field is slowly turned on. In the region $b > 0$ the BPS energy level has crossed to positive energy, so what was previously the ground state of the Dirac sea (described by curve (a)) is reinterpreted as the state with a single physical fermion with energy proportional to b, indicating the presence of the string. Likewise, the hole state (curve (b)) is reinterpreted as the new second-quantized ground state. The presence of the two curves that are related by mirror reflection is essential for this interpretation, and the interchange symmetry is a manifestation of the CPT symmetry of the second-quantized IIA description.

Type IIA string theory in the background of a D6-brane and a transverse D2-brane has no other finite-energy BPS states, beyond those described by the above two curves. To see that no other holomorphic curves are possible in M-theory consider a more general curve $F(w, v) = 0$. The degree of F considered as a polynomial in w determines the number of D2-branes oriented in the (x_7, x_8) directions in the string theory limit. This is the number of solutions of the equation for fixed v. Since we want to describe a single D2-brane we will take the degree to be one, so that the curve must have the form

$$w = C(v) \; . \tag{9}$$

However, we also require that the membrane is at finite x_9 for all finite values of v. Therefore, since w only diverges at $x_9 \to -\infty$, the meromorphic function $C(v)$ cannot have poles in the complex v-plane. Such poles would correspond to semi-infinite strings attached to the D2-brane in the type IIA limit and stretching to the left.

Consider next the zeros of w, which can appear either at $x_9 \to \infty$ and any v, or at $v = 0$ and any x_9. The first correspond to semi-infinite strings stretching to the right, in the type IIA limit, and are not allowed. Therefore, the most general form for $C(v)$ is $C = av^n$. But the form (4) of the w-coordinate shows that $|w|$ vanishes *at most* as fast as $|v|$ at the origin. The only allowed values are therefore $n = 0, 1$. This proves the claim that there are only two holomorphic curves, with no infinite strings attached, and they correspond precisely to the two quantum states discussed above.

Of course, it is very easy to find surfaces that are not holomorphically embedded and, in the IIA limit, describe two or more fundamental strings joining a D6-brane and a D2-brane. However, these are not BPS states and break supersymmetry. The ground state of such a configuration would define a stable non-supersymmetric vacuum state of a quantum field theory living in the membrane. In the IIA limit the mass gap between the lowest BPS string state and excited string states is of order $\hbar(2\pi\alpha')^{-1/2}$. It would be interesting to determine the energy of such an M-theory configuration directly, but we have not investigated this issue. The system studied here transforms, through a chain of dualities, to the system considered in [1] consisting of a D5-brane and a NS 5-brane. These may be joined by a single BPS D3-brane but states with multiple D3-branes are not supersymmetric. This is the s-rule that is in accord with the absence of supersymmetric vacua for three-dimensional $U(k)$ supersymmetric Yang–Mills theory with Fayet-Iliopoulos term for $k > 1$. It might be of interest to see whether this mechanism can be generalized to apply to supersymmetry breaking in more realistic contexts.

Acknowledgments

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